EECS C106B / 206B Robotic Manipulation and Interaction **Spring 2020**

Lecture 17: Introduction to Grasping

Scribes: Saagar Sanghavi, Jewook Ryu, Ahan Sabharwal

17.1 Overview and History

Recall that the elbow manipulator was inspired by the human arm, and it has a 2DOF base (shoulder), 1DOF elbow, and 3DOF wrist attached to the end effector. When the elbow manipulator has equal-length upper and lower arm lengths, the dextrous workspace is maximized.

Multi-fingered hands are another popular robotic manipulator that was inspired by the human hand. Human hands provide us the ability to perform fine motion, manipulate objects, interface with the external world, and grasp objects.

The history of hand design traces back to the first models for prosthetic devices. As the field of robotics emerged, hand-like structures were used as dextrous end end effectors. The progress of artificial hand design often required in-depth study of human hands to seek inspiration from biology. We know that 30-40% of the human motor cortex in the brain is used for hand control—similarly, the design of hand-like manipulators is one of the most challenging parts of robotics.

How should artificial hands be manipulated? One approach is to control the joints of a hand directly using motors—however, this approach has proven to be hard to control as there is lots of friction, and also quite "whimpy" and not able to provide enough force to the fingers. Another approach is to use a system of pulleys and strings, inspired by tendons in human hands. This approach has the drawback that the strings may become "stretchy" over time, they may get frayed, and other issues. Hydraulic controls (ie. using pipes with fluid under pressure that is manipulated) is a third approach—care must be taken to not use harmful hydraulic fluids in certain situations, like when involved in a robot-assisted surgery inside a human body.

The key design issues in designing hands are challenges involving mechanical systems, sensors/actuators, and control hardware.

17.2 Grasp Contact Models

Contact models can be characterized by their Force vectors, $\vec{F}_i$. These vectors include generalized forces and torques along each axis.

Frictionless Point contact (FPC)

$$\vec{F}_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x$$

where $x \in \mathbb{R}$

Point Contact with Friction (PCWF)

$$\vec{F}_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{x}_i$$

where $\vec{x}_i \in FC_i = \{\vec{x}_i \in \mathbb{R}^3 : \sqrt{x_{i1}^2 + x_{i2}^2} > \mu_i x_{i3}\}$, and μ_i is the coefficient of friction

Soft Finger Contact (SFC)

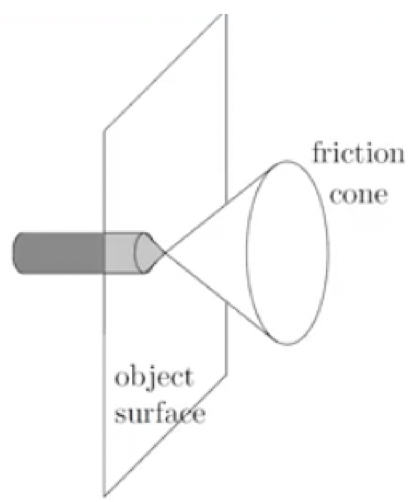
$$\vec{F}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \vec{x}_i$$

where $\vec{x}_i \in FC_i$. Here we have two models for FC_i, the elliptic model and the Linear Model. Note that μ_i is the coefficient of linear friction and μ_{it} is the torsional coefficient of friction.

Elliptic Model: $FC_i = \{\vec{x}_i \in \mathbb{R}^4 : \sqrt{\frac{1}{\mu_i^2}(x_{i1}^2 + x_{i2}^2) + \frac{1}{\mu_{it}^2} x_{i4}^2} \leq x_{i3}\}$

Linear Model: $FC_i = \{\vec{x}_i \in \mathbb{R}^4 : \frac{1}{\mu_i}\sqrt{(x_{i1}^2 + x_{i2}^2)} + \frac{1}{\mu_{it}}|x_{i4}| \leq x_{i3}\}$

A Grasp can be characterized by its contact models and its object frame.



Soft Finger Contact

17.3 The Grasp Map/Matrix G

The grasp map/matrix G tells you the relationship between the wrench exerted on the contact point and the wrench on the body center of mass.

Recall that in 106a, we spent a lot of time transforming twists from body to spatial frame and back, enabled by the matrix $G = (R, p)$. We can view the grasp matrix in a similar way, where p is the distance from the body center of mass to the contact point and R is the relative rotation between the body coordinate frame and contact coordinate frame.

17.3.1 Single Finger in FPC

This is the force exerted by a single finger in frictionless point contact:

$$F_0 = \begin{bmatrix} R_{c_i} & 0 \\ \hat{p}_{c_i} R_{c_i} & R_{c_i} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_i = \begin{bmatrix} n_{c_i} \\ p_{c_i} \times n_{c_i} \end{bmatrix} x_i$$

where $x_i \geq 0$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ are the torque entries. The torque gets rotated by R_{c_i}. If $R_{c_i} = I$ and n_{c_i} was the normal vector of contact in the body frame, then $R_{c_i} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ becomes the normal vector n_{c_i}. The $p_{c_i} \times n_{c_i}$ term applies the force and converts it into the body frame. You transform it into the body frame because the body is going to move around.

The matrix $G = \begin{bmatrix} R_{c_i} & 0 \\ \hat{p}_{c_i} R_{c_i} & R_{c_i} \end{bmatrix} \in \mathbb{R}^{6\times 6}$, which transforms the applied wrench into the body frame, gives a normal vector and a p vector crossed with the normal vector because the point of application of force gets transferred.

17.3.2 K Fingers in FPC

If you have any such k fingers stacked next to each other, F_0 instead becomes

$$F_0 = \begin{bmatrix} n_{c_1} & \dots & n_{c_k} \\ p_{c_1} \times n_{c_1} & \dots & p_{c_k} \times n_{c_k} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix} = Gx$$

where $F_0 \in \mathbb{R}^6, x \geq 0$.

Let's go through an example:

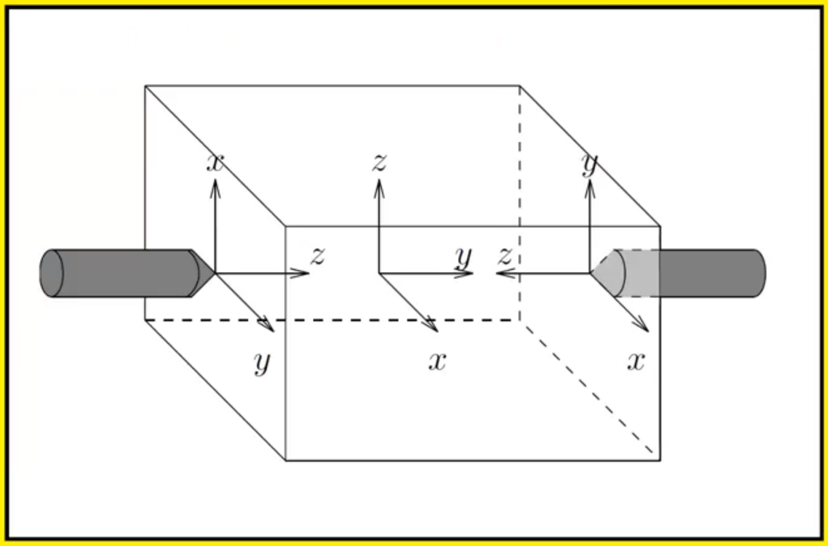
17.3.3 Example: Box Being Held by 2 Soft Fingers

$$R_{c_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},$$

$$p_{c_1} = \begin{bmatrix} 0 \\ -r \\ 0 \end{bmatrix},$$

$$R_{c_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$p_{c_2} = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix},$$



$$G_i = \begin{bmatrix} R_{oc_i} & 0 \\ \hat{p}_{oc_i} R_{oc_i} & R_{oc_i} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here's an example of a box being held by 2 soft fingers, c_1 to the left and c_2 to the right. The resulting calculations are as follows:

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & r & 0 & 0 & -r & 0 & 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} & x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} \end{bmatrix} \in \mathbb{R}^8$$

$$FC = FC_1 \times FC_2$$

$$FC_i = \left\{ x_i \in \mathbb{R}^4 \,\middle|\, \sqrt{\frac{1}{\mu_i}(x_{i,1}^2 + x_{i,2}^2) + \mu_{it} x_{i,4}^2} \leq x_{i,3}, x_{i,3} \geq 0 \right\}, i = 1, 2$$

Here, the first column of G (from the left) is the normal force, the next two are the friction forces, and the fourth column is the torque. We have four columns associated to each finger. Note we use the elliptic model for soft finger contact.

17.4 Force Closure

We can think of a grasp as being stable when the grasper is able to exert force to immobilize a held object when there is a force present that tries to pull the object away. This notion is formalized by force closure.

Definition: A grasp (G, FC) is **force closure** if $\forall F_0 \in \mathbb{R}^p, \exists x \in FC$ such that $Gx = F_0$.

There are usually 3 main problems we try to solve:

Force Closure Problem: Determine if a grasp (G, FC) is force-closure or not.

Force Feasibility Problem: Given $F_0 \in \mathbb{R}^p$, determine if $\exists x \in FC$ such that $Gx = F_0$.

Force Optimization Problem: Given $F_0 \in \mathbb{R}^p$, find the $x \in FC$ such that $Gx = F_0$ and x minimizes some objective function $\Phi(x)$

We can define an **internal force** as: $x_N \in FC$ if $Gx_N = 0$ or $x_N \in (kerG \cap FC)$. For example, 2 fingers exerting equal forces in opposite directions on an object. The net force will be 0, this is an internal force.

Property 3: (G,FC) is force closure $\iff$ $G(FC) = R^p$ and $\forall x_N \in ker$ G s.t. $x_N \in int(FC)$.

17.4.1 Solutions of the force-closure and force-feasibility relationships (Linear algebra review)

Proposition: Linear Matrix Inequality (LMI) Property

Given $Q(x) = S_0 + \sum_{l=1}^{m} x_l S_l$, where $S_l = S_l^T$, $l = 0, 1, \ldots, m$, the sets $A_Q = \{x \in \mathbb{R}^m | Q(x) \geq 0\}$ and $B_Q = \{x \in \mathbb{R}^m | Q(x) > 0\}$ are convex.

LMI Feasibility Problem

Determine if the set A_Q or B_Q is empty or not.

We can use linear programming tools to check whether there are solutions to $Gx = F_0$ given x even if x lies inside the friction cone. In this case you would be approximating the friction cone, which is a pyramid and therefore a quadratic surface, as a LMI, and then checking for the solutions of the LMI. All of the aforementioned problems can be transformed into LMI's.

17.4.2 Constructing Force-Closure Grasps

Theorem 1: Planar antipodal grasp

A planar grasp with two point contacts with friction is force-closure iff the line connecting the contact point lies inside both friction cones.

Antipodal means separable by a hyperplane.

Once you converted grasp planning into a linear algebra question (LMIs in this case), you can use all of the machinery of optimization to be able to give bounds on the number of fingers that are needed. Below are the bounds on the number of fingers required to grasp different types of objects:

Space	Object type	Lower	Upper	FPC	PCWF	SF
Planar ($p = 3$)	Exceptional	4	6	n/a	3	3
	Non-exceptional			4	3	3
Spatial ($p = 6$)	Exceptional	7	12	n/a	4	4
	Non-exceptional			12	4	4
	Polyhedral			7	4	4

17.5 Professor Sastry's summary of the lecture

It's really about force closure: solving $Gx = F_0$, the number and locations of of x's. That's the hot topic in grasping.