EECS C106B / 206B Robotic Manipulation and Interaction **Spring 2020**

Lecture 15: Uncalibrated Geometry & Stratification

Scribes: Abanob Bostouros Adith Sundram

15.1 Learning Objectives

1. Uncalibrated epipolar geometry
2. Pre-calibration with partial/full scene knowledge

15.2 Definitions

- Real world reference point X = $[x, y, z, w]^T \in R^4$, (w = 1)
- Image plane coordinates $\chi = [x, y, 1]^T$
- Camera extrinsic parameters g = (R, T)
- Perspective projection $\lambda x = [R, T]X$
- Pixel coordinates $x' = K x$
- Projection matrix $\lambda x' = \Pi X = [KR, KT]X$
- $K = \begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_\theta & o_y \\ 0 & 0 & 1 \end{bmatrix}$, where f is the focal length, s_x, s_y s_θ is the distortion

15.3 Taxonomy on Uncalibrated Reconstruction

- K is known, back to $x = K^{-1} x'$
- K is partially known, we can use parallel lines, vanishing points, planar motion, and constant intrinsic.
- K is completely unknown,
  - Estimate calibration with complete scene knowledge
  - Reconstruct despite the lack of K
  - Recover from uncalibrated images

15.4 Uncalibrated Epipolar Geometry

We see the following Linear Transformation to Relate the Calibrated and Pixel Coordinates for a given camera and frame

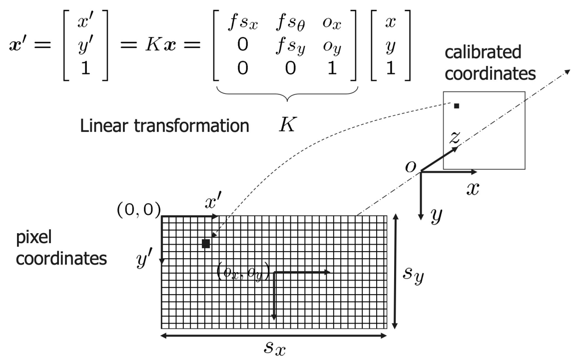

Figure 15.1: Sinusoid Planner Simple Motion

15.5 The Fundamental Matrix F

F is a nonzero matrix F $\in \mathbb{R}^{3\times3}$ is a fundamental matrix if $(SVD)F = U\Sigma V^T$

$$\Sigma = diag\{\sigma_1, \sigma_2, 0\} for \sigma_1, \sigma_2 \in R_+$$
$$det(F) = 0$$

$$F = K^{-T}\hat{T}RK^{-1} = \hat{T}'KRK^{-1}$$

Note that F is Rank 2 because T is not invertable

15.6 Estimating F

- Find F that minmizes epipolar error over pixel cordinates

$$min_F \sum_{j=1}^{n} x_2'^{jT} F x_1'^{j}$$

$$a = x_1' \otimes x_2'$$
$$F^s = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9]$$
$$a^T F^s = 0$$
$$\chi f^s = 0$$

15.7 Two View Linear Algorithm

Solve LLSE Problem

$$min_F \sum_{j=1}^{n} \mathbf{x}_2'^{jT} F \mathbf{x}_1'^{j} \Rightarrow \chi F^s = 0$$

Find the Solution Eigenvector associated with the smallest Eigenvalue

Compute SVD of F recovered from data

$$F = U\Sigma V^T \quad \Sigma = diag(\sigma_1, \sigma_2, \sigma_3)$$

We can then project onto the Manifold

$$\Sigma' = diag(\sigma_1, \sigma_2, 0) \quad F = U\Sigma' V^T$$

15.8 Calibration with a Rig

Given 3-D Coordinates on Known object X, we can eliminate scale, two linear components per point

$$\begin{aligned} x^i(\pi_3^T \mathbf{X}) &= \pi_1^T \mathbf{X}, \\ y^i(\pi_3^T \mathbf{X}) &= \pi_2^T \mathbf{X} \end{aligned}$$

We can then recover the following Projection Matrix

$$\Pi^s = [\pi_{11}, \pi_{21}, \pi_{31}, \pi_{12}, \pi_{22}, \pi_{32}, \pi_{13}, \pi_{23}, \pi_{33}, \pi_{14}, \pi_{24}, \pi_{34}]^T$$
$$\min \|M\Pi^s\|^2 \quad \text{subject to} \quad \|\Pi^s\|^2 = 1$$

and solve for the translation

Solve for Homography from the Plane to the Image

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = K[r_1, r_2, T] \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Two linear constraints on the calibration per image

$$H \doteq K[r_1, r_2, T] \quad \in \mathbb{R}^{3\times 3} \quad K^{-1}[h_1, h_2] \sim [r_1, r_2]$$

$$h_1^T K^{-T} K^{-1} h_2 = 0, \quad h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2.$$

Note that pre-fabricated objects like cube are not practical as their fabrication must be perfefct and calibrated which is an expensive and unreliable way of calibration

15.9 Calibration with a Planar Rig

Leveraging the fact that 2-D coordinates of feature points on a pre-fabricated plane are known. we can use it instead of the 3-D object (eg. cube) to calibrate

15.10 Calibration with Scene Structure

15.10.1 Vanishing points

Intersection of Orthogonal Directions

$$\mathbf{v}_1 = KRe_1, \quad \mathbf{v}_2 = KRe_2, \quad \mathbf{v}_3 = KRe_1$$

Since for Orthogonal Directions, the Inner Product is Zero, we can thus solve for the Constraints on the Matrix S

15.10.2 Calibration with Motions: Pure Rotation

Uncalibrated two views related by a pure rotation:

$$\lambda_2 K\mathbf{x}_2 = \lambda_1 KRK^{-1}K\mathbf{x}_1 \quad \widehat{\mathbf{x}'_2}KRK^{-1}\mathbf{x}'_1 = 0$$

We have the following linear constraints

$$S^{-1} - CS^{-1}C^T = 0 \text{ where } S^{-1} = KK^T$$