A

EECS C106B / 206B Robotic Manipulation and Interaction **Spring 2022**

Lecture 14A: (Introduction to Legged Robots)

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14.1 History of legged robots

14.1.1 Motivation

The motivation for developing legged robotics is to ultimately mic animal function. The human world is designed for legged locomotion, so legged robots have distinct advantages over traditional locomotion methods. Legged robots allow for:

- Increased agility
- Increased mobility
- Can be used for prosthetics

14.1.2 Challenges

However, legged robots also have a unique set of challenges that make it hard to design a real time controller for. These include:

- Many degrees of freedom
- Under actuation/nonlinear models
- Balance
- Rugged terrain
- Periodic versus static equilibrium

14.1.3 Notable Firsts

The first known legged ”robot” were walking horse mechanisms driven by humans like a bike. As there were no motors and computers, control and actuation were carried out by the rider.

The first computer controlled legged robots were designed by Raibert’s Leg Lab, which eventually became Boston Dynamics. They developed bipedals, hopping and quadruped robots in the late 90s.

14.1.4 Cassie

Cassie is a bipedal robot used by Professor Sreenath's lab. They identified a key issue with legged robots: They are very inefficient when walking on flat ground compared to wheels. Their solution was to develop a multi-modal transport method for Cassie, by developing a controller to help it locomote using hover shoes.

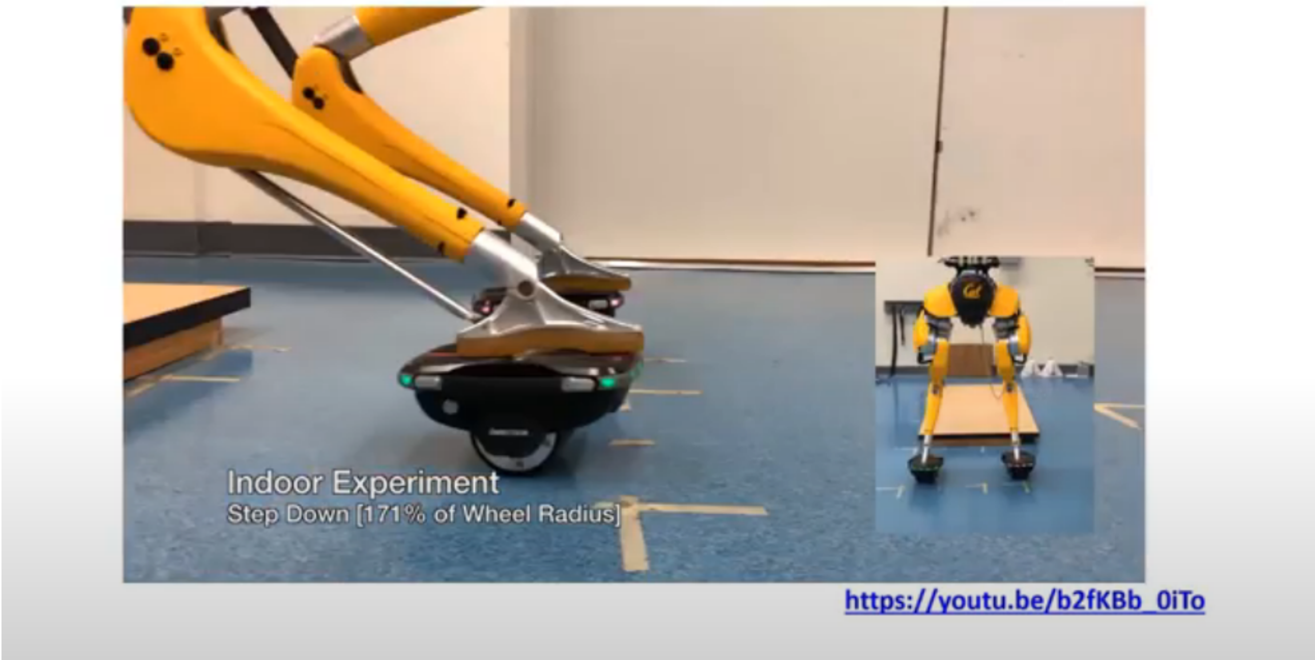

Figure 14.1: An image of Cassie balancing on Hover shoes.

14.1.5 Cassie Juggling

Dynamical Model of Ball:

$$m_b \ddot{x}_b = -m_b g e_3 + R f_b \tag{14.1}$$

Dynamical Model of Paddle:

$$m_p \ddot{x}_p = -m_p g e_3 - R f_b + f \tag{14.2}$$

$$J_p \dot{\Omega} = -\Omega \times J_p \Omega - R^T (x_b - x_p) \times f_b + M \tag{14.3}$$

When you add these together, you get a combined controller with the following equation:

$$D(q)\ddot{q} + H(q,\dot{q}) = Bu + J_s^T(q)\tau_s + J_c^T f_{feet} + J_b^T(q) f_b \tag{14.4}$$

Dynamical Model of Ball Bouncing:

In free fall:

$$m\ddot{y} = -mg \tag{14.5}$$

This model of the ball changes upon impact due to the forces applied on it, The bouncing model then becomes:

$$m\ddot{y} = -mg, y > 0 \tag{14.6}$$

$$\begin{bmatrix} y^+ \\ \dot{y}^+ \end{bmatrix} = \Delta(\begin{bmatrix} y^- \\ \dot{y}^- \end{bmatrix}) \tag{14.7}$$

Where Δ maps the state before and after impact. The simplest model is to sue the coefficient of restitution to relate the velocity before and after impact.

14.2 Dynamics Model of a Legged Robot

- Continuous-time Model
- Impact Model
- Hybrid Model

14.2.1 Continuous-time Model

- Lagrangian: $\mathscr{L}(q,\dot{q}) := K(q,\dot{q}) - V(q)$
- Euler-Lagrange Eq.: $\frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{q}} - \frac{\partial \mathscr{L}}{\partial q} = \Gamma$
- Robot Dynamics: $D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Gamma$
  $K(q,\dot{q}) = \frac{1}{2}\dot{q}^T D(q)\dot{q}$
  $c_{kj} = \sum_{i=1}^{\bar{N}} \frac{1}{2}(\frac{\partial D_{kj}}{\partial q_i} + \frac{\partial D_{ki}}{\partial q_j} - \frac{\partial D_{ij}}{\partial q_k})$
  $G(q) = \frac{\partial V(q)}{\partial q}$
- Generalized Force (Force at a point): $\Gamma_i = (\frac{\partial p_i}{\partial q})^T F$

In these equation, the robot dynamics is described by Lagrange equation of motion. In the above equations, q is the state (joint angle) of the robot, K denotes kinetic energy of the robot, and V denotes the potential energy of the robot. The rests can be described these three terms.

14.2.2 Impact Model

In this model, the state q and $dotq$ are combined and denoted as x. Besides, to deal with the impact dynamics, we should consider that dynamics in the end effector frame. There are some points to consider the impact model.

- An impact results from the contact of the swing leg end with the ground.
- The impact is instantaneous.
- The impact results in no rebound and no slipping of the swing leg.

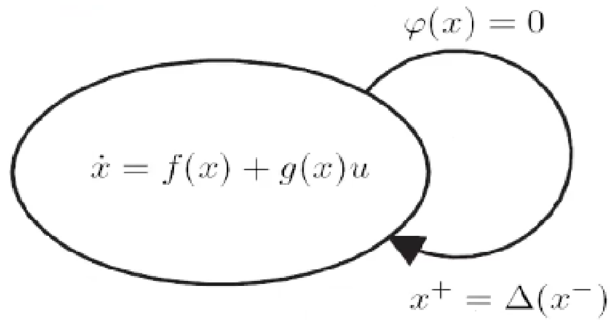

Figure 14.2: Schematic diagram of the hybrid model. The relationship between the continuous model and impact model is described.

- In the case of walking, at the moment of impact, the stance leg lifts from the ground without interaction, while in the case of running, at the moment of impact, the former stance leg is not in contact with the ground.
- The externally applied forces during the impact can be represented by impulses.
- The actuators cannot generate impulses and hence can be ignored during impact.
- The impulsive forces may result in an instantaneous change in the robot's velocities, but there is no instantaneous change in the configuration.

The impact model is given by a delta function.

$$x^+ = \Delta(x^-) \tag{14.8}$$

$$x^- = [q^-, \dot{q}^-]^T \tag{14.9}$$

$$x^+ = [q^+, \dot{q}^+]^T \tag{14.10}$$

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Gamma + \delta F_{ext} \tag{14.11}$$

where - notation means before impact and + notation means after impact. Then, we integrate over duration of impact ($t^- \to t^+$).

$$D(q^+)\dot{q}^+ - D(q^-)\dot{q}^- = F_{ext} \tag{14.12}$$

$$F_{ext} = \int_{t^-}^{t^+} \delta F_{ext}(\tau) d\tau \tag{14.13}$$

Finally, the constraint becomes

$$J_{sw}(q^+)\dot{q}^+ = 0 \tag{14.14}$$

where F_{ext} denotes the external force during the impact and J_{sw} denotes the Jacobian of a swing foot.

14.2.3 Hybrid Model

In Fig. 14.3, the dynamics in each phase is described by the continuous-time model and the dynamics between the two phases is described by the impact model. By combing the continuous-time model and impact model, we can describe robots' running and walking motion.

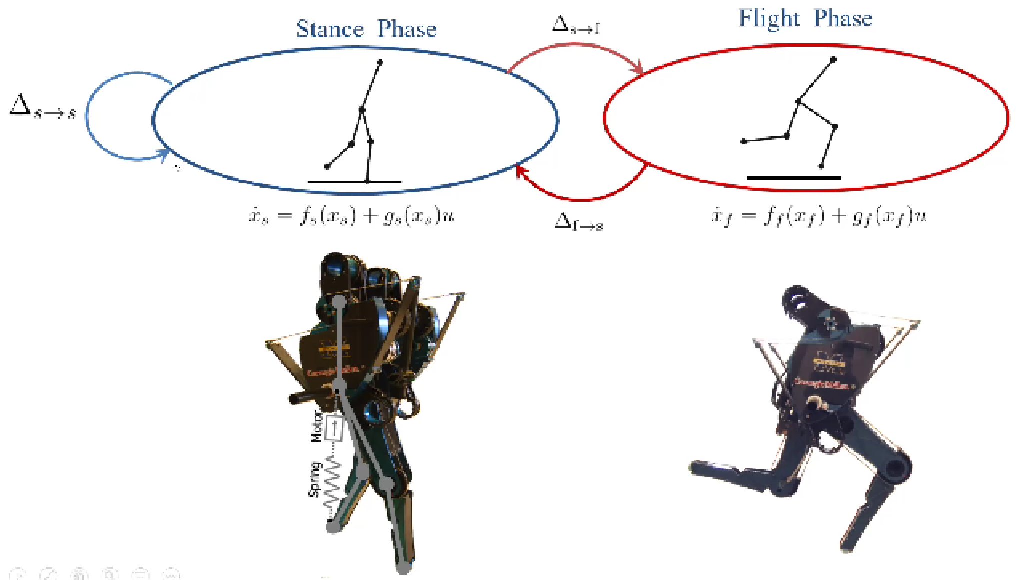

Figure 14.3: Hybrid model for running of a legged robot. Since we should consider two phases, stance phase and flight phase, the impact model should be considered in addition to the two continuous dynamics.

14.3 How to Make Robots Walk and Run

The overview of legged locomotion approaches is shown in the following figure. At first, we consider controllers for legged locomotion.

14.3.1 Zero Moment Point (ZMP)

First of all, we consider simplified biped model displayed in Fig. 14.5. The assumptions are

- Running cart of massless table
- Cart represents the CoM motion
- Table represents supporting foot

. If the position of projected CoM is outside of the supporting leg and the cart does not move, the table starts rotating. However, if the cart is moving, then the torque of the friction force between the cart and the table makes the table rotate in the opposite direction. Hence, the rotation of the table can be cancelled. In the controller, we try to find such torque that can cancel the rotation.

ZMP considers similar thing to this. Figure 14.6 shows the trajectory of the center of pressure on the foot. During legged locomotion, the heel firstly contacts with the ground and finally toe touches the ground. So the trajectory starts from the heel and then ends at the toe. ZMP controls that the entire trajectory lies on the foot (In this case, we consider the case of a foot, so the support polyhedron is a surface of the foot.). If the center of pressure reaches the boundary of the foot, the foot should rotate. In Fig. 14.7, the legged locomotion of a flat-footed robot is described. During the fully actuated period, we assign the control input, torque T_d and force F_d to the robot. F_{Gnd} and F_{Fric} are the normal and friction forces, respectively due to the contact with the ground and we assume these reaction forces act at a point. This

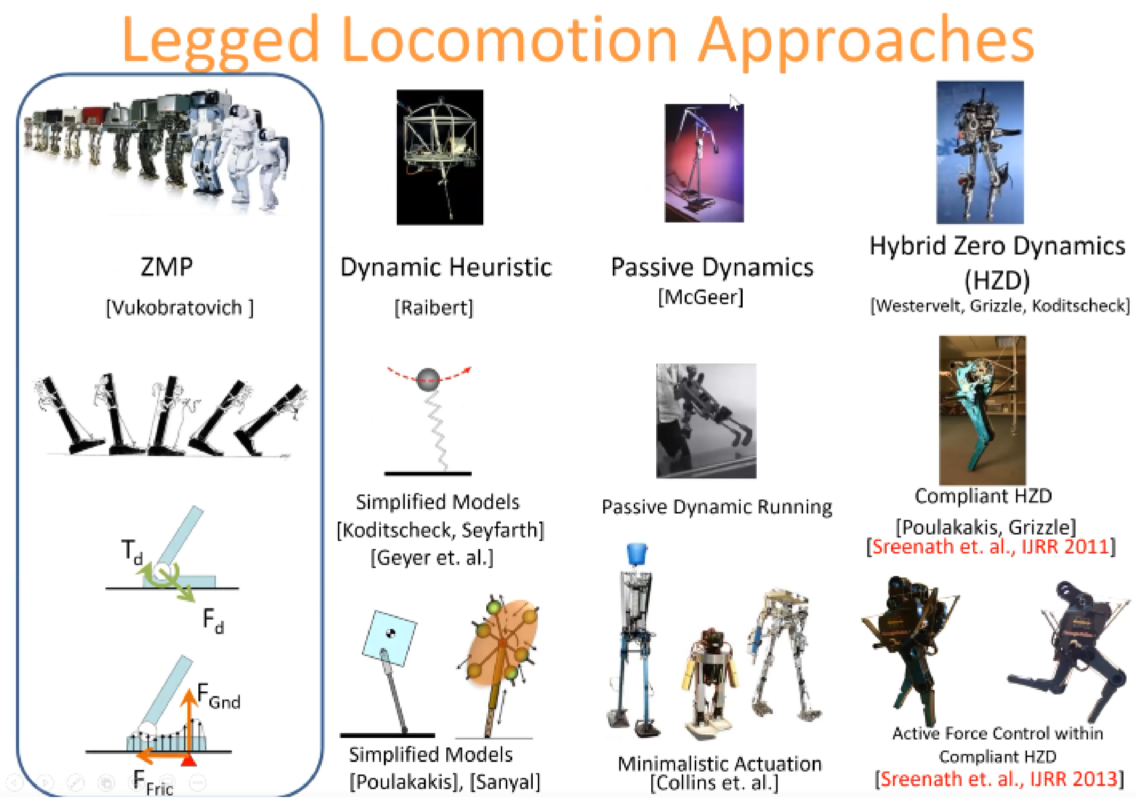

Figure 14.4: Hybrid model for running of a legged robot. Since we should consider two phases, stance phase and flight phase, the impact model should be considered in addition to the two continuous dynamics.

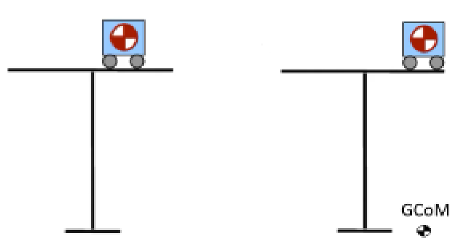

Figure 14.5: Simplified biped model. The black lines are simplified legs and the cart is the center of mass (CoM) of legged robot. The relationship between the legs and cart is described.

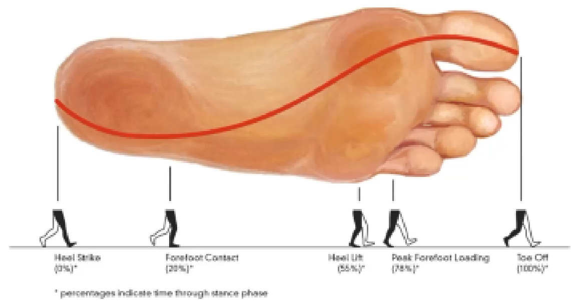

Figure 14.6: Center of pressure of a leg. The red line on the foot is trajectory of center of pressure. In ZMP control, we make the entire trajectory lie in the foot.

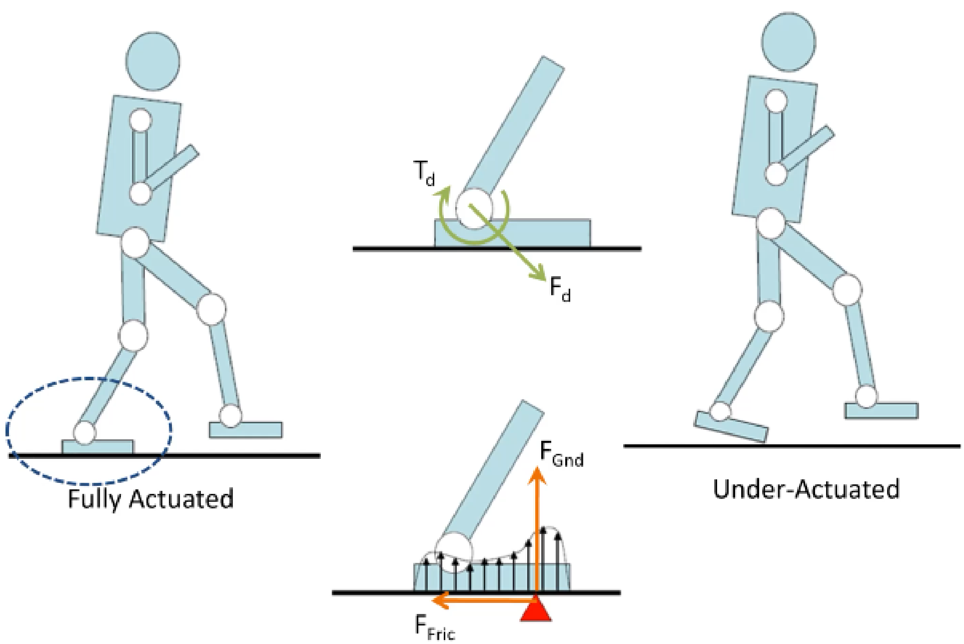

Figure 14.7: Procedure of the flat-footed walking. If the flat foot is entirely in contact with the ground, the state is called fully actuated. However, if a part of the foot is in contact with the ground, that state is called under actuated.

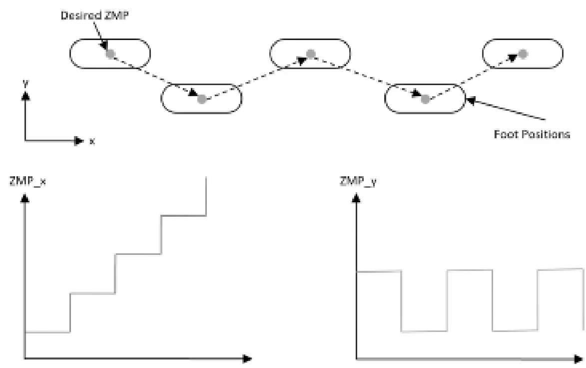

Figure 14.8: Feet positions of bipedal walking. The ovals are the foot positions with desired ZMP. The bottom two figures show the time profile of the positions of the foot.

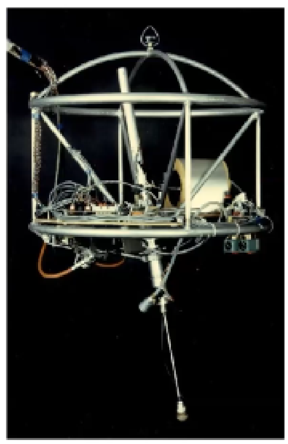

Figure 14.9: An example of a hopping robot. By shrinking the foot, the robot realizes hopping motion.

point is center of pressure. By changing the control inputs, we make the center of pressure entirely lie in the support polyhedron. Figure 14.8 shows an example of bipedal walking with desired ZMP.

14.3.2 Raibert Control

Raibert control is a heuristic control approach. This approach considers dynamic control during under actuated phases. In this lecture, a simple hopping robot is introduced as an example (See Fig.**??**). To maintain the hopping motion, the controller should address these three things.

- Maintain the hop height
- Maintain the body attitude
- Maintain the hopping speed

At first, we consider the hopping height. To maintain the hopping height, we address how much force should be

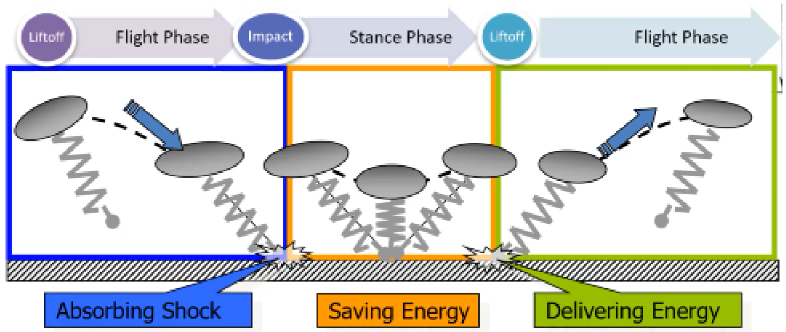

Figure 14.10: Procedure of a hopping motion. During the contact, the robot saves the energy and utilizes the energy to flight.

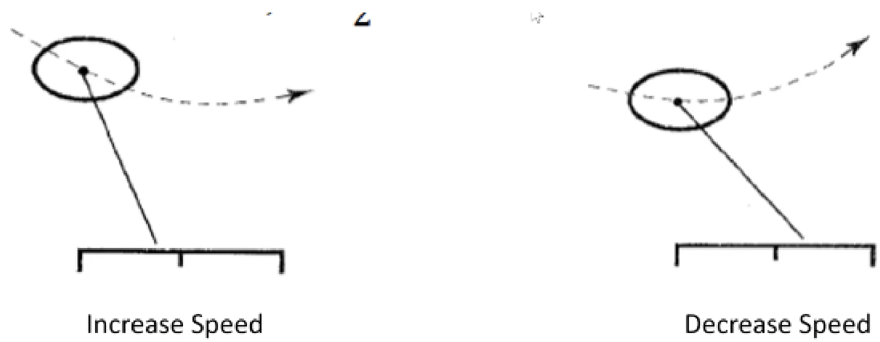

Figure 14.11: Relationship between foot position and the body velocity. If the foot position is backward the desired contact point, the velocity of the robot is lower than the desired velocity. If the foot position is forward the desired contact point, the velocity of the robot is higher than the desired velocity.

applied to the robot. Then, we utilize the following equation.

$$\tau_{spring} = k_p(h_{des} - h_{k-1}) \tag{14.15}$$

where τ_{spring} is a control input, k_p is a proportional gain, h_{des} is the desired peak height, and h_{k-1} is the actual peak height of the previous hop. Figure 14.10 shows the procedure of a hopping motion.

Secondly, we address maintaining the body attitude.

$$\tau_{hip} = k_p(\phi_{des} - \phi) + k_v(\dot{\phi}_{des} - \dot{p}hi) \tag{14.16}$$

where τ_{hip} is hip torque, k_p is a proportional gain, ϕ_{des} is desired body attitude, ϕ is actual body attitude, k_v is a differential gain, $\dot{\phi}_{des}$ is desired body angular velocity, and $\dot{\phi}$ is actual body angular velocity. By utilizing this PD controller, we can maintain the body attitude of the robot.

Thirdly, we address maintaining the hopping speed.

$$x_f = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_{des}) \tag{14.17}$$

where x_f is foot position, $\dot{x}$ is velocity of the robot, T_s is a period of stance phase, and $k_{\dot{x}}$ is a differential gain. By utilizing this controller, we can find appropriate foot position of the robot. As shown in Fig.14.11, if the foot position is behind the desired contact point, the controller increases the velocity of the robot. On the other hand, if the foot position is forward the desired contact point, the controller decreases the velocity of the robot.

In the case of two legs, one of the legs is idle while the other leg is in contact with the ground, hence, we can apply the Raibert controller to the two legs robot. In the case of four legs, two of them are synchronized, hence we can consider the motion of two legs. Therefore, we can also apply the Raibert controller to four legged robot.