Discussion 7 : Two-View Geometry

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7.1 Question 1: Camera Projection Matrices

Consider the point X^{in} expressed with respect to the world coordinate system (or equivalently with respect to first camera coordinate system). Suppose that the transformation between the two cameras is described by the rotation matrix R and the translation vector t so that $X^{in\prime} = RX^{in} + t$.Assume that the first camera calibration matrix is K and that the second camera calibration matrix is K'. What are the camera projection matrices P and P' that allow us to write:

$$x = PX$$

$$x' = PX'$$

7.2 Question 2: Epipoles

Write the expression of the epipoles e and e'.

7.3 Question 3: Recap

Consider two identical cameras (such that $K = K'$), whose optical axis coincides with the z axis, whose focal length is 0.03 m, with square pixels, no pinhole point offset and zero skew. What is the expression of K? Hint: K is the camera calibration matrix here.

7.4 Question 4: Understanding the Epipolar Line

What is the projective interpretation of $e' \times x'$? How does this object relate to the epipolar line l'?

7.5 Question 5: Essential Matrix

Given the that first camera's 3D nodal point can be represented with d^L and the second camera's 3D nodal point can be represented with d^R. Let R^L be the rotation from the global frame of the first camera and R^R the same for the second camera. How can you represent the Essential Matrix for this setup?

7.6 Question 6: Fundamental Matrix

Let:

$$F = \begin{bmatrix} f_1 & f_4 & f_7 \\ f_2 & f_5 & f_8 \\ f_3 & f_6 & f_9 \end{bmatrix}$$

and let $\mathbf{f} = [f_1, ..., f_9]$. Show that the epipolar constraint can be represented as the following inner product:

$$a(x_i, x_i')\mathbf{f} = 0$$

where $a(x_i, x_i')$ is a row vector that depends only on the coordinates of the point x_i and x_i'

Note: $x_i' F x_i = 0$

7.7 Question 7: Fundamental Matrix Details

Answer the following questions, providing an adequate justification.

1. How many degrees of freedom does the fundamental matrix F have?
2. How many independent equations do we need to estimate f? Why?
3. Explain how the system of equations can be written in matrix form as Af = 0. What are the dimensions of A? Write explicitly A.
4. How would you estimate f via constrained least squares?

7.8 Question 8: One more Time - Fundamental Matrix

Recall that the fundamental matrix can be written as $F = \hat{e}'K'RK^{-1}$. Show that $e' \times x' = Fx$.