EECS C106B / 206B Robotic Manipulation and Interaction **Spring 2022**

Discussion 6

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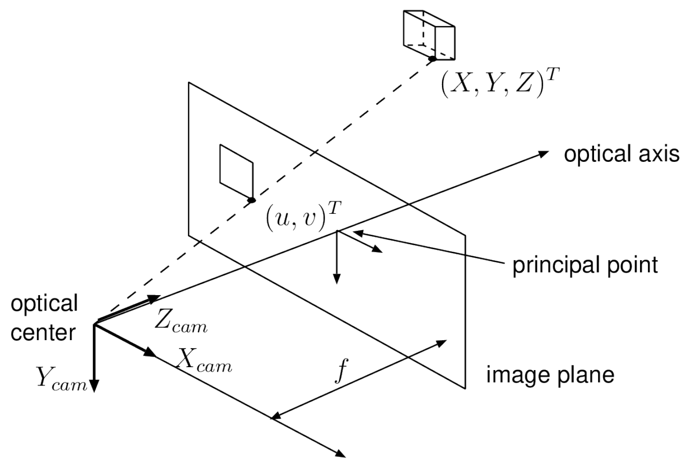
6.1 Question 1: Pinhole Model



Using the image above, find the relationship between the 3D point $(X, Y, Z)^T$ to its corresponding 2D projection (u, v) on to the imaging plane.

Hint: Use Law of Similar Triangles.

$$u = f(x/z)$$

$$v = f(y/z)$$

6.2 Question 2: Camera Intrinsic Matrix

The camera intrinsic parameter matrix K is represented as

$$\begin{bmatrix} fs_x & s_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

What do these terms represent?

The camera intrinsic parameter matrix K can be represented

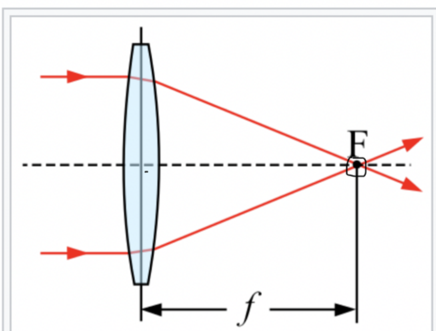
$$\begin{bmatrix} fs_x & s_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1.1}$$

→ normally
Ox = Image_width/2
Oy = Image_height/2

focal length

What do each of these terms represent?

http://ksimek.github.io/2013/08/13/intrinsic/



In practice, f_x and f_y can differ for a number of reasons:

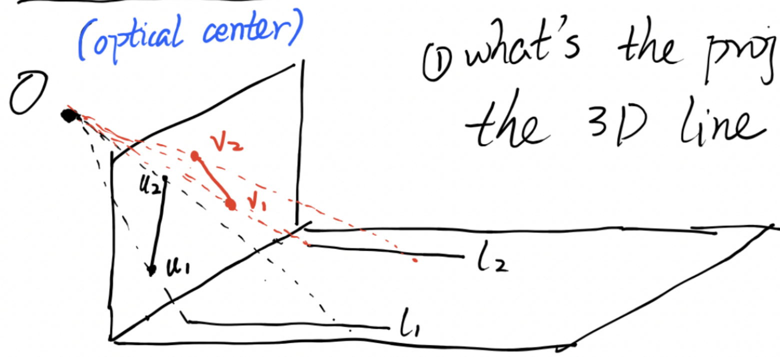
- Flaws in the digital camera sensor.
- The image has been non-uniformly scaled in post-processing.
- The camera's lens introduces unintentional distortion.
- The camera uses an anamorphic format, where the lens compresses a widescreen scene into a standard-sized sensor.
- Errors in camera calibration.

6.3 Question 3: Vanishing Points

A straight line in the 3D world becomes a straight line in the image. However, two parallel lines in the 3D world will often intersect in the image. The point of intersection is called the vanishing point.

1. Given two parallel lines, how do you compute the vanishing point?
2. When does the vanishing point not exist (the two lines do not intersect)?
3. Show that the vanishing points of lines on a plane lie on the vanishing line of the plane.

1) Given two lines in an image, and you know they are parellel in 3D.
Then what is the direction of this 3D lines?



① what's the projection of the 3D line on image plane?

$\Rightarrow (u_1, u_2)$

$\Rightarrow (V_1, V_2)$

$u_1, u_2, V_1, V_2 \in R^2$

Consider the normal vector of $\triangle O u_1 u_2$, $\triangle O V_1 V_2$

$$\vec{n}_{\triangle O u_1 u_2} = \overrightarrow{Ou_1} \times \overrightarrow{Ou_2}$$
$$\vec{n}_{\triangle O V_1 V_2} = \overrightarrow{OV_1} \times \overrightarrow{OV_2}$$

$$\begin{cases} \vec{d} \perp \vec{n}_{\triangle O u_1 u_2} \\ \vec{d} \perp \vec{n}_{\triangle O V_1 V_2} \end{cases} \quad \boxed{\vec{d} = \vec{n}_{\triangle O V_1 u_2} \times \vec{n}_{\triangle O V_1 V_2}}$$

direction of l_1, l_2

2) when $\vec{d}$ // image plane

3) ① what is vanishing line of a plane?

Assume $N = (N_x, N_y, N_z)$ is the norm of plane.

$P = (X, Y, Z)$ is a point in the plane.

$\Rightarrow N^T P = d$ (plane equation, d is a constant)

Consider P's projection on image plane

$\left(\begin{array}{l}\text{recall the pinhole camera model} \\ x = f\frac{X}{Z},\ y = f\frac{Y}{Z}\end{array}\right)$

$\Rightarrow x N_x + y N_y + f N_z = \frac{fd}{Z}$ (x, y is Projection of P)

Let $Z \to \infty$, we get a line in image plane

$\vec{l} = x N_x + y N_y = -f N_z$

② For any line in the plane, its vanishing point lies in $\vec{l}$

For any line in the plane

$\vec{a} + \lambda \vec{d}$ $\begin{cases} \vec{a} = (a_x, a_y, a_z) \\ \vec{d} = (d_x, d_y, d_z) \\ N^T d = 0 \end{cases}$

The vanishing point is:

$\left(\frac{f(a_x + \lambda d_x)}{a_z + \lambda d_z}, \frac{f(a_y + \lambda d_y)}{a_z + \lambda d_z}\right)$

$\overset{\lambda \to \infty}{=} \left(f\frac{d_x}{d_z}, f\frac{d_y}{d_z}\right)$ substitude in the plane equation

$N_x d_x + N_y d_y + N_z d_z = 0$

$\Updownarrow$

$\underline{f\frac{d_x}{d_z} N_x + f\frac{d_y}{d_z} N_y = -f N_z}$