

# EE106A Discussion 3: Forward Kinematics

## 1 Forward kinematics

### 1.1 Numbering notation for joints and links

We use the following notation for joints and links: Number the links from 0 to  $n$  starting from the base. Then, number the connecting joints such that joint  $i$  connects links  $i - 1$  and  $i$ . Typically, we attach the *base frame*  $S$  to be stationary with respect to link 0, and the *tool frame*  $T$  to the robot end-effector.

**Definition 1.** The *joint space*  $Q$  of a manipulator is composed of all possible values of the joint variables of the robot. This is equivalent to the configuration space of the robot. Each joint is parameterized by its *joint angle*  $\theta$ , even though both angles and displacements are allowed depending on the joint type (revolute or prismatic).

### 1.2 Forward kinematics problem statement

The forward kinematics of a robot determines the configuration of the end-effector (the gripper or tool mounted on the end of the robot) given the relative configurations of each pair of adjacent links of the robot. Thus, the main objective of forward kinematics is finding the transformation  $g_{st}(\theta_1, \theta_2, \dots, \theta_n)$  as a function of the joint angles  $\theta_1 \dots \theta_n$ .

### 1.3 Product of exponentials formula

Given the initial configuration  $g_{st}(0)$  when all joint angles are at 0, we can use the twist  $\hat{\xi}_i$  associated with each joint  $i$  and compose them to get the resulting configuration  $g_{st}(\theta)$  as a function of  $\theta_1 \dots \theta_n$  which we call  $\theta$  here.

$$g_{st}(\theta) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} \dots e^{\hat{\xi}_n \theta_n} g_{st}(0) \quad (1)$$

Recall the twist for a revolute joint where  $\omega$  is a unit vector in the direction of the twist axis, and  $q$  is any point on the axis:

$$\xi = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} \quad (2)$$

and the twist for a prismatic joint, where  $v$  is a unit vector in the direction of translation:

$$\xi = \begin{bmatrix} v \\ 0 \end{bmatrix} \quad (3)$$

**Problem 1.** Does the composition work in any other order in general? Why or why not?

## 2 SCARA forward kinematics

**Problem 2.** Find the forward kinematics map for the manipulator shown in Fig. 2.

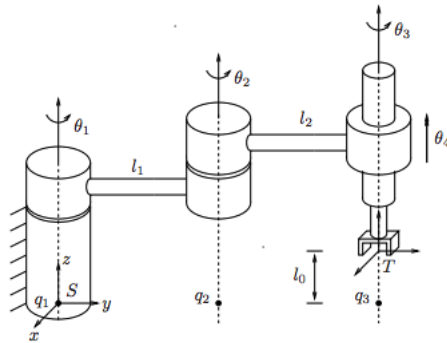


Figure 1: SCARA manipulator in its reference configuration (joint angles all 0).

**Problem 3.** *Show that you can arrive at the same result using rigid body transformations in non-exponential coordinates.*

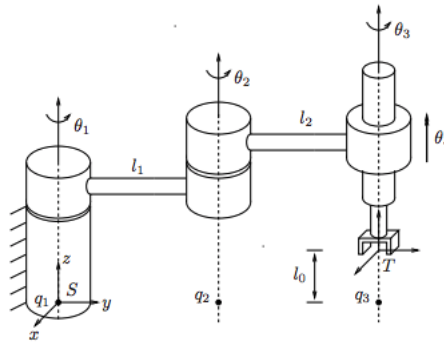


Figure 2: SCARA manipulator in its reference configuration (joint angles all 0).

### 3 Elbow manipulator forward kinematics

**Problem 4.** Find the forward kinematics map for the elbow manipulator in Fig. 3.

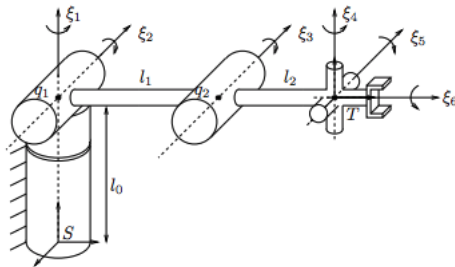


Figure 3: Elbow manipulator in its reference configuration (joint angles all 0).